We assume that the other characteristics are also straight, constant properties lines. (A detailed computation of the actual flow would show that the foregoing two assumptions, in the geometric considerations that follow, have, to a certain extent, balancing effects.)

As in the two-dimensional case, we write the continuity equation across a typical characteristic (in Ref. 3 a straight line perpendicular to the actual plug profile was instead selected).

The surface crossed by the flow is

$$S = 2\pi \frac{r_e + r}{2} \frac{r_e - r}{\sin\alpha} \tag{1}$$

Since the velocity makes an angle μ with the surface, the actual passage area is

$$A = S \sin\mu = \frac{\pi(r_e^2 - r^2)}{M \sin\alpha} \tag{2}$$

The length of the characteristic from the nozzle lip to the plug surface is

$$l = \frac{r_e - r}{\sin\alpha} \tag{3}$$

whereas the exit area is

$$A_e = \pi(r_e^2 - r_b^2)$$

Equation (3), taking into account Eq. (2), may be written

$$l = \frac{r_e - [r_e^2 - (AM \sin\alpha/\pi)]^{1/2}}{\sin\alpha}$$

or, in a nondimensional form,

$$\xi = \frac{l}{r_e} = \frac{1 - \{1 - [\epsilon(1 - \eta_b^2)M \sin\alpha/\epsilon_e]\}^{1/2}}{\sin\alpha} \tag{4}$$

in which η_b represents the nondimensional base radius.

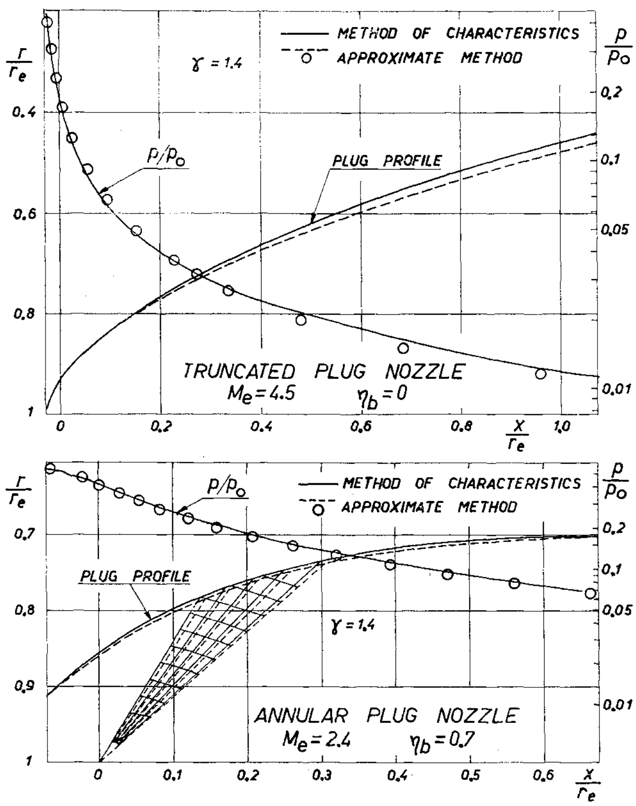

Fig. 3 Comparison of approximate and exact solutions in plug nozzle design.

The equations of the plug profile in a parametric form are then

$$\begin{aligned} \xi &= \xi(M) \\ \alpha &= \nu_e - \nu(M) - \mu(M) \end{aligned} \tag{5}$$

in which the Mach number varies from 1 to the design exit value M_e.

Figure 2 shows several plug profiles computed in this way for various Mach numbers and base radii.

In Fig. 3, shapes and pressure distributions of two isentropic plugs, computed by the method of characteristics, are compared with the results of the approximate method.

As a consequence of our assumptions, the agreement between the two design techniques is good wherever the expansion characteristics do not depart significantly from straight, constant properties lines.

This, in fact, occurs for nozzles having a large base radius, no matter what the exit Mach number, and for nozzles with a high exit Mach number in the supersonic region just downstream of the throat, no matter what the base radius.

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Plasma Radiation Shielding

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It has been recognized for some time that energetic protons constitute a serious radiation hazard in space, especially for trips lasting longer than a week or two. An important attribute of the radiation shielding problem is that very few methods are available to us for dealing with it. This note describes an approach to the problem which has not to our knowledge been suggested before. At this stage, the new approach indicates the possibility of a substantial reduction in the weight of a space radiation shield.

Three methods of shielding are currently available. First, of course, there is solid shielding. Second, pure magnetic shielding has been shown to have substantial advantages over solid shielding, but only for very large vehicle sizes.[1–3] Third, there is electrostatic shielding[4] in which the space vehicle to be protected must be kept at a positive potential of 1 or 2 × 10^8 v relative either to an outer part of the space vehicle or to "infinity." Maintaining a potential difference of this order of magnitude between two solid conductors is well beyond the limit of present-day technology using heavy ground equipment. On the other hand, the electrons present in the interplanetary plasma would rapidly discharge any positive potential of the whole space vehicle relative to infinity. The power required to maintain a potential of 2 × 10^8 v against this loss is estimated to be about 10^7 kw.

If it were possible to reduce very substantially the flow of electrons from space to the vehicle, electrostatic shielding might, after all, be feasible. Our suggestion is based on the fact that under suitable conditions electrons do not flow

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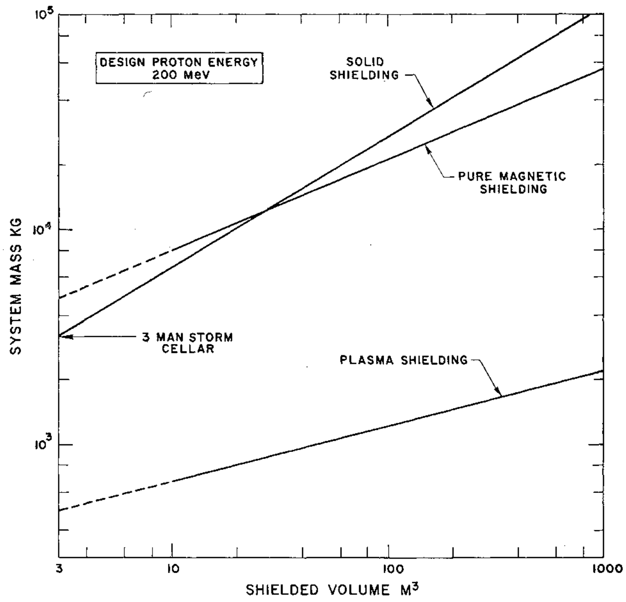

Fig. 1 Comparative weights for different shielding systems. The weights of the pure magnetic shield are taken from Refs. 3 and 4. The plasma radiation shield weight includes an allowance for a high-energy electron accelerator that may not be necessary. At lower design proton energy, plasma shielding shows an even greater advantage over pure magnetic shielding.

across magnetic field lines. Thus, in the presence of a magnetic field strong enough to control the electron motions, a space vehicle might be maintained at a very high potential relative to infinity. The magnetic field necessary to control electrons of modest energy is far less than that required to control very energetic protons as in the pure magnetic shielding scheme. As a result, the device as a whole is far lighter than the pure magnetic shield. A preliminary comparison of the weights of the various systems is given in Fig. 1. This scheme, which we call plasma radiation shielding, shows an even greater advantage over pure magnetic shielding at lower values of the design proton energy. Plasma radiation shielding has two principal physical requirements. The first requirement is for a lightweight means of charging the vehicle. The second requirement is for an effective mechanism of

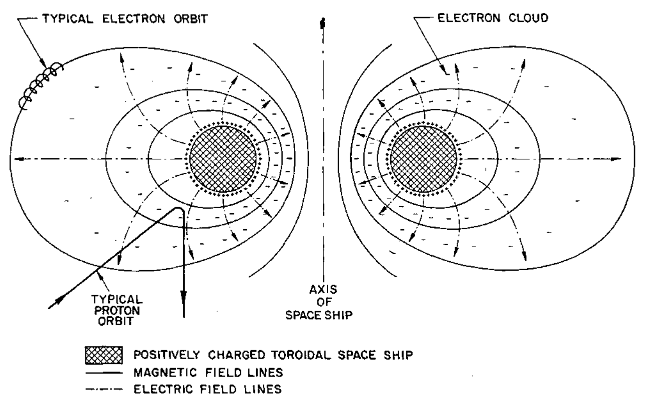

Fig. 2 Principle of the plasma radiation shield. The shield is basically electrostatic and repels protons by virtue of being positively charged. Electrons, attracted by the positive charge, cannot cross the magnetic field lines and discharge the vehicle. Instead, they drift azimuthally around the vehicle as shown in Fig. 3. Viewed in a frame moving with the drift velocity, they also execute helical (thermal) motions around the field lines as shown here. The total charge in the electron cloud is equal and opposite to the charge on the vehicle.

"containing" electrons away from the vehicle. Fortunately, our containment mechanism also provides the basis for a remarkably simple charging scheme. As a consequence of this, primary consideration must be given to containment problems.

The way in which the electrons are restrained from flowing to the vehicle requires that the shape of the magnetic field be such that no field line that extends a long way from the vehicle should intersect the surface of the vehicle. This consideration drives us to the toroidal shape illustrated schematically in Figs. 2 and 3. There is, however, some latitude in the design of the cross section, and the four-coil arrangement shown is no more than a suggestion.

In the presence of crossed electric and magnetic fields, electrons acquire a drift motion with the velocity $(\mathbf{E} \times \mathbf{B})/B^2$, provided that the magnitude of this quantity is less than the speed of light. Since E is determined by the design proton energy and the over-all size of the space vehicle, the condition $E/B < c$ gives a lower limit to the strength of the magnetic field, namely, E/c. If B is much greater than E/c, we approach a pure magnetic shield and could dispense with the electric field. We do not at present know how close to this minimum value of B we can design, but we have assumed provisionally that we can work with $B \approx 2\,E/c$. The direction of the electron drift is azimuthally around the space vehicle. In addition to the drift motion, the electrons can be expected to have thermal motions; a typical thermal motion is illustrated in Fig. 2.

Knowledge of the required magnetic and electric fields allows us to calculate all those quantities appropriate to the plasma radiation shield which are independent of the magnitude of the losses in the device and do not pertain to the starting of the shield. Rather than do this by means of a series of formulas, we present in Table 1 a list of design parameters of the type referred to, based on a design proton energy of 200 Mev and a major radius for the shield of 5 m. The derivation of all the quantities given in the table follows from these two design numbers and the assumption that $E/B = c/2$. The total positive charge on the vehicle is obtained by integrating the normal component of the electric field over the surface area of the vehicle. The electron cloud, which is distributed on the magnetic field lines in the neighborhood of the shield, must have an equal and opposite negative charge. This observation allows us to calculate the electron density in the neighborhood of the shield.

We observe that no ions can be trapped in the magnetic field since their Larmor radii would typically be larger than the size of the magnetic field, and they are therefore promptly

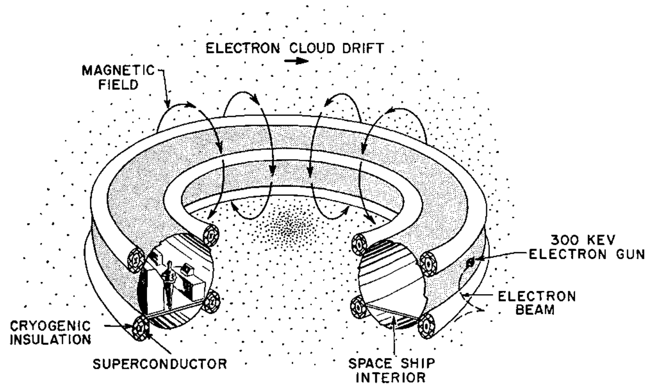

Fig. 3 Schematic diagram of a space vehicle using a plasma radiation shield. The four-coil superconducting magnet system shown here is no more than a suggestion. The charge ejection system shown utilizes the inductive mechanism associated with the turning on of the magnet in order to transport electrons away from the vehicle. Ejection from the vehicle must be accomplished at a velocity greater than E/B velocity, or about 300 kev.

ejected by the electric field. The absence of ions together with the relatively large electron density leads to the very striking observation that we are dealing here with a "one-component plasma." The use of the word plasma is justified by noting that the electric field in which each electron moves is determined by the instantaneous positions of all the other electrons so that the electrons cannot be considered as following independent trajectories in fixed external fields, but must be treated as strongly correlated. No one-component plasma of this type has (to our knowledge) ever been produced in the laboratory or discussed theoretically. Only the rather special geometry under consideration here appears to make it feasible. We do not propose to discuss the properties of this plasma here, but two observations are of particular interest.

Table 1 Typical plasma radiation shield quantities independent of the magnitude of the losses

Assumed quantities		
Over-all voltage	V_0	2×10^8 v
Size	R	5 m
Derived quantities		
Electric field	E	4×10^7 v/m
Magnetic field	B	2660 gauss
Cyclotron frequency	$\omega_c/2\pi$	7 k Mc
Circumferential drift velocity	v_{drift}	1.5×10^8 m/sec
Total charge	Q	0.3 coul
Number of electrons	N_e	2×10^{18}
Electron density	n_e	2×10^9 cm^{-3}
Plasma frequency	$\omega_p/2\pi$	400 Mc
Magnetic field energy	U_M	1.2×10^8 joules
Electric field energy	U_E	3×10^7 joules
Power required to charge both fields in 5000 sec		30 kw
Electron drift energy		80 kev
Electron flux		3×10^{19} cm^{-2} sec^{-1}
Momentum flux		4×10^2 dynes/cm^2
Energy flux		4×10^5 w/cm^2
Magnet current	I_{magnet}	2×10^6 amp
Mass of superconductor	M_{sc}	550 kg
Cryogenic area	A_{cry}	10 m^2
Cryogenic mass	M_{cry}	180 kg
Cryogenic power	P_{cry}	6 kw

First, each electron moves in such a way as to satisfy roughly the equation

$$\mathbf{E} + \mathbf{v}_e \times \mathbf{B} = 0$$

for motions having characteristic frequencies less than cyclotron frequency. Multiplying this equation by $n_e e$ we find

$$n_e e\, \mathbf{E} = \mathbf{j} \times \mathbf{B}$$

In other words, the plasma is in equilibrium under the opposing electric and magnetic body forces. The electron pressure is less than both these forces by a factor on the order of kT/eV_0, where kT is the mean electron energy measured in a frame moving with the drift velocity, and V_0 is the design electrostatic potential of the shield. This equilibrium is to be contrasted with the more usual situation in plasma physics in which charge neutrality prevails, $\mathbf{E}$ is small, and the Lorentz force is balanced by the plasma kinetic pressure. In connection with the electron plasma, we also note that we can calculate a Debye length h, but that the interpretation of this length is not the usual one. It is now the size of the region which contains sufficient electrons to make a potential kT/e at its surface. From this definition we find that the ratio of h to R, the size of the shield, is

$$h/R \approx (kT/eV_0)^{1/2} \ll 1$$

It is this observation that ultimately justifies our description of the electron cloud as a plasma.

As in all machines that, in the steady state, have charged particles moving in trapped orbits, injection is a problem. In our case, two alternative methods of injection are suggested. Of these, the more desirable consists simply of injecting the electrons onto field lines close to the vehicle while the magnetic field is being built up. As the magnetic field increases, field lines initially near the space vehicle move away from it, carrying with them any electrons that happen to be on them. The resulting separation of charge sets up the electric field. This scheme suffers from the limitation that, if for any reason losses occur which reduce the electrostatic potential of the device (and therefore the energy in the electric field), they can be made good only by increasing the strength of the magnetic field. Therefore, the scheme is only usable if the losses are so low that the electric field does not leak away in a time shorter than the time for which shielding is needed. If we are thinking in terms of solar flares, the electric field should stay on by itself for a day or two. If the losses are somewhat greater than this, it will be

Table 2 Typical plasma radiation shield quantities dependent on the magnitude of the losses

Assumed loss:	500 w[a]	10 kw[b]
Corresponding radial leakage current	2.5×10^{-6} amp	5×10^{-5} amp
Corresponding radial leakage current density	6×10^{-13} amp/cm^2	1.25×10^{-11} amp/cm^2
1) If these losses are due entirely to outgassing[c]:		
Acceptable outgassing rate	$<1.5 \times 10^{13}$ atoms/sec	$<3 \times 10^{14}$ atoms/sec
Average outgassing rate	$<5 \times 10^6$ atoms/cm^2 sec	$<10^8$ atoms/cm^2 sec
Acceptable surface vapor pressure	$<5 \times 10^{-14}$ mm Hg	$<10^{-12}$ mm Hg
2) If these losses are due entirely to micrometeorites[c]:		
Acceptable flux (iron)	$<1.5 \times 10^{-8}$ g/cm^2-yr	$<3 \times 10^{-7}$ g/cm^2 yr
3) If these losses are caused by any form of diffusion in the electron gas:		
Electron containment time $= Q/I_{\text{leakage}}$	$>1.2 \times 10^5$ sec	$>6 \times 10^3$ sec
Radial drift velocity, $v_{\parallel E}$	$<4 \times 10^{-5}$ m/sec	$<8 \times 10^{-4}$ m/sec
Drift Angle $= (1/\omega_e\tau_e)_{\text{eff}} = j_{\parallel E}/j_{\perp E}$	$<3 \times 10^{-13}$	$<6 \times 10^{-12}$
"Effective" collision time with fixed centers	>80 sec	>4 sec
Electron temperature (thermal energy)	<80 kev	<1 Mev
Electron gyro radius	<3 mm	<1 cm
Debye length	<4 cm	<15 cm

[a] Initial inductive charge ejection, no further charge ejection necessary after start-up.
[b] Needs continuous operation of linac during flares.
[c] Assuming worst case, i.e. each atom ionized at the surface.

necessary to eject electrons continuously during a solar flare in order to maintain the electric field. This could be done with an electron linear accelerator emitting a beam of electrons at the design energy. These electrons could be expected to escape through the magnetic field on account of the relativistic increase in their mass. An allowance for this linear accelerator has been made in the weights quoted in Fig. 1. However, because of the greater simplicity and negligible weight of the inductive charge ejection scheme, it is to be hoped that the accelerator would be unnecessary.

It can be seen from the foregoing discussion that the magnitude of the losses that we can expect is at present unknown. Losses take the form of motion of electrons toward the space vehicle or positive ions away from the vehicle, in either case at the expense of the energy of the electric field. Taking first the losses due to positive ions, we note that, since there are no trapped ions in the system, and since ions coming from outside the system (including the solar flare ions) are reflected without loss, the only source is from the ionization of neutral atoms in the electric field region. Following such an ionization, the electron that is born is retained on the magnetic field line where it is, but the ion is simply ejected. The worst case is if ionization takes place at the surface of the vehicle, for then each ion ejected carries with it an energy eV_0 acquired from the electric field. Two possible sources of neutral atoms are outgassing from the surface and micrometeorites. Assuming pessimistically that each ionization is at the surface of the vehicle, Table 2 shows the permissible outgassing rates for two values of the power consumed by the ion current. The outgassing can be seen to represent a serious problem, but it is probably not insuperable since conditions in space are very favorable to achieving a good bakeout of exposed surfaces. The micrometeorite rate near the earth is given by Whipple[5] as about 10^{-6} g/cm^2-yr, but evidence obtained in deep space by Alexander,[6] using an instrument aboard Mariner II, showed flux rates 10^4 times lower than corresponding rates near the earth.

The remaining source of loss arises from the possibility of diffusive motion of the electrons toward the space vehicle. This source of loss is at present, by many orders of magnitude, the least certain aspect of the whole device. We are attempting here to confine a plasma with a magnetic field; experience gained in the field of controlled thermonuclear fusion prompts us to comment on this problem with extreme caution. We can, however, point out that our configuration having the magnetic field "inside" and the plasma "outside" does fulfill the so-called minimum B requirement presently thought to contribute to stability.[7] Furthermore, certain types of instabilities which might have been expected to contribute to substantial rates of diffusion across the magnetic field,[8] and which are thought basically to be a result of the difference between the masses (and hence mobilities) of electrons and ions, will in our case be absent. We can also show that electron-electron collisions will give rise to a classical diffusion, which can be shown to be negligibly small. On the other hand, the confinement has to be very good indeed for our device to work. In the lower part of Table 2 we list the requirements on the containment process. We note that each electron is required to circle the device some 10^{12} times in the drift (azimuthal) direction before diffusing across the magnetic field. The maximum permissible temperature is calculated by assuming that the loss power heats the electron gas and that the heat thus gained is lost by cyclotron radiation.

These remarks on losses lead us to conclude on this note: The plasma radiation shield, as presently conceived, violates no principle of physics. On the other hand, it requires a degree of plasma containment greatly in excess of anything hitherto achieved. Although the configuration and other factors appear favorable, no definite answer will be possible without experimental verification. The range of uncertainty is at present so great that marginal operation seems unlikely.

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Approximate Solution of Second-Order Boundary-Layer Equations

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Nomenclature

k = dimensionless curvature of the body surface
l = reference length
n, s = dimensionless coordinates, normal to and along the body surface
N = $n\, Re^{1/2}$
Re = Reynolds number $= ul/\nu$
u = reference velocity
$U^{(1)}, U^{(2)}$ = dimensionless first- and second-order tangential velocity components, evaluated at thebody surface

$$\alpha^{(1)} = \int_0^\infty (U^{(1)} - \psi_N{}^{(1)})dN$$

$$\alpha^{(2,d)} = \int_0^\infty (U^{(2)} - \psi_N{}^{(2,d)})dN$$

$$\alpha^{(2,k)} = -\int_0^\infty (kU^{(1)}N + \psi_N{}^{(2,k)})dN$$

$$\alpha^{(2,\Omega)} = -\int_0^\infty (\Omega N + \psi_N{}^{(2,\Omega)})dN$$

$$\beta^{(1)} = \int_0^\infty (U^{(1)2} - \psi_N{}^{(1)2})dN$$

$$\beta^{(2,d)} = \int_0^\infty (U^{(1)}U^{(2)} - \psi_N{}^{(1)}\psi_N{}^{(2,d)})dN$$

$$\beta^{(2,k)} = -\int_0^\infty (kU^{(1)2}N + \psi_N{}^{(1)}\psi_N{}^{(2,k)})dN$$

$$\beta^{(2,\Omega)} = -\int_0^\infty (\Omega U^{(1)}N + \psi_N{}^{(1)}\psi_N{}^{(2,\Omega)})dN$$

$$\gamma^{(1)} = \int_0^\infty (U^{(1)} - \psi_N{}^{(1)2})N\, dN$$

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